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RELIABILITY GROWTH MODELING

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Mathematical models describing the changes in system reliability during a development program are called "reliability growth" models. The intent of this report is to familiarize the reader with reliability growth modeling and to discuss its usage as a tool for program managers. The discussion is supplemented by numerical examples illustrating several models. Also, a review of a number of reliability growth models currently available and a bibliography on reliability growth are given.

PREFACE

The government's materiel acquisition process for new systems requiring development is invariably complex and difficult for many reasons. Generally the materiel systems involve appreciable departures from existing systems common to the commercial world. The development of the materiel systems frequently involve new technology and a challenge to the state of the art. The pressure to improve combat effectiveness invariably requires compressed time scales as compared to similar developments in the commercial world. The requirements for reliability and performance parameters are usually highly demanding and the combat and climatic environments invariably cover broad spectrums. Procurement actions, of necessity, must adhere to uncommon constraints, e.g. lowest bid. The exercises in controls are also somewhat constrained. The procedures for management of resources, funding, manpower, contracting and time scales present formidable management tasks. Vendor-vendee relationships are somewhat different from those of the commercial world. The requirements, their development, coordination and communication present dynamic and difficult problems.

It is, therefore, essential that we continually strive for improvement in the management of the materiel acquisition process. One avenue for improvement which is currently in a formative or emergent stage is that of <u>reliability growth modeling</u>. Reliability growth modeling has the potential to accomplish several things:

a. To place into prospective the relationship of requirements with respect to stage of materiel acquisition.

b. To focus attention on the need for quantitatively tracking reliability throughout the materiel acquisition stages.

c. To focus attention on failures to meet goals in order that corrective action may be taken promptly.

d. To aid in the allocation and reallocation of resources to achieve goals on schedule and within constraints.

e. To establish a logical basis for projecting reliability growth so that this consideration may properly be included in the decision making process.

Most often there is not a meeting of the minds as to the stage of development where the requirements stated in Materiel Needs (MN's) documents are to be met. It is believed that most of the requirements are generated with the thought in mind that they represent what the user desires of the materiel at the time it is fielded or during field use. Current practice seems to overlook this and assumes that the requirement is to be met at all stages of the materiel acquisition and test process. Frequently, decisions at all levels are affected while these anomalies exist. It is therefore quite important that management tools be designed to recognize the transition in reliability growth throughout the materiel acquisition and test process and that interim goals be established accordingly. This should be of considerable assistance in achieving a sounder management and decision process during the materiel acquisition life cycle. The other aforementioned points are, also, of great importance because the failures to quantitatively track materiel reliability most often leads to appalling surprises, consternation and costly, untimely decisions. Management's degrees of freedom, quality of decision, and alternatives are highly dependent on quantitative measures of where we are and where are we going as often as practicable throughout the life cycle. Corrective actions must be based on adequate knowledge of the reliability and performance of the materiel systems during milestones of the materiel acquisition process and the timely reorientation of resources can be an important ingredient in effective management.

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1. INTRODUCTION AND SUMMARY

A topic of considerable interest in current reliability studies is that of accounting for the changes in system reliability that result from design and engineering modifications during a development program. A development program is generally recognized as being a necessity for most systems since they usually exhibit initial design and engineering deficiencies. Attempts are made during the development program to find and remove these deficiencies to a point where certain levels of performance with respect to reliability and other requirements are met.

It is usually assumed that the system reliability will increase during the development program and, thus, mathematical models describing this phenomenon have come to be called "reliability growth" models. The purpose of this report is to acquaint the reader with the benefits of reliability growth modeling as a tool for program managers and to familiarize him with some of those models and their applications.

Specifically, in Section 2 reliability growth modeling is discussed as a management tool for program managers. Some background on the general area of reliability growth modeling is given in Section 3 and in Section 4 several models are illustrated by numerical examples.

Section 5 discusses briefly certain practical considerations which should be taken into account when choosing a reliability growth model. Appendix A presents a review of a number of reliability growth models currently available and Appendix B is a bibliography on reliability growth.

2. RELIABILITY GROWTH MODELING - A MANAGEMENT TOOL

Although the purpose of a development program for a system is to increase the system's reliability to a level acceptable to the user, it is unfortunately true that many programs fail to achieve this goal within initial cost, manpower and time limitations. The major cause for these failures is associated with the complexities of many of today's systems which make it extremely difficult to determine before development the relationships between the required final reliability, the system's basic design and the total development effort. As a result, the program managers may be faced with the responsibility of planning a development program without firm rationale as to how to allocate the various resources and how to determine the project milestones.

The purpose of the project milestones is to set goals which will guide the development program step by step so that the reliability requirement will be met at least within the final stage of development. However, if the necessary development effort is not known beforehand then the program managers must try to determine whether or not the milestones are being met as the development program progresses. If, for example, the reliability of the system at the present milestone is found to be below a specified level then additional development effort may be necessary to meet the required level at the next milestone. Furthermore, given the results of the development program up to the present time the program managers may question the realism of meeting the reliability requirements of future milestones with present resources.

Therefore, in light of various uncertainties the program managers must try to estimate the progress of the development program up to the present time and relate this by some means to the future development of the system. In general, however, it is often a difficult task for one to obtain good estimates of the progress of the development program and project future progress. For example, to directly estimate the system's

reliability at some time during development, a certain number of systems, which are homogeneous with respect to the design configuration at that time, must be tested. The number of systems tested would depend on the precision desired in the estimate. In many cases this approach is extremely costly and may even be impossible if the needed prototypes cannot be produced. Also, it should be remarked that the proportion of successful tests of the system up to the present time measures only the average reliability of past tests and does not measure the present reliability or aid in predicting the future reliability of the system. The average reliability will lie between the initial and final reliability of the system in most cases.

If the progress of the development program cannot be reasonably estimated then it is clear that the program managers may have difficulty planning and conducting the program to insure that the required reliability will be met as scheduled. It is apparent, too, that program managers generally need specialized techniques and methodology in this regard. The area of reliability growth modeling is a management tool directed toward this need of the program managers.

3. BACKGROUND

The development of a system generally evolves as a repeated process of system examination and testing, determination of system failure modes, and design and engineering changes as attempts to eliminate these modes. The problem of accounting for the resulting changes in the system's reliability during a development program by mathematical models has been of interest for a number of years. Much of the research on this subject, however, has not been published in the open literature and is often difficult to obtain.

The central purpose of most reliability growth models includes one or both of the following objectives:

- Inference on the present system reliability;
- Projection on the system reliability at some future development time.

Most of the reliability growth models considered in the literature assume that a mathematical formula (or curve), as a function of time, represents the reliability of the system during the development program. It is commonly assumed, also, that these curves are non-decreasing. That is, once the system's reliability has reached a certain level, it will not drop below this level during the remainder of the development program. It is important to note that this is equivalent to assuming that any design or engineering changes made during the development program do not decrease the system's reliability.

If, before the development program has begun, the exact shape of the reliability growth curve is known for a certain combination of system design and development effort, then the model is a deterministic one. In this case the amount of development effort needed to meet the reliability requirement could be determined, and the sufficiency of the design would, also, be known.

In most situations encountered in practice, the exact shape of the reliability growth curve will not be known before the development program begins. The program manager may, however, be willing to

assume that the curve belongs to some particular class of parametric reliability growth curves. This is analogous to life testing situations when the experimenter assumes that the life distribution of the items is a member of some parametric class such as the exponential, gamma, or Weibull families. The analysis then reduces to a statistical problem of estimating the unknown parameters from the experimental data. These estimates may be revised as more data are obtained as the development program progresses. Using these estimates, the program manager can monitor and project the reliability of the system and make necessary decisions accordingly.

Some Bayesian reliability growth models have, also, appeared in the literature. This approach assumes that the unknown parameters of the growth curve are themselves random variables governed by appropriate prior probability distributions. Generally, the form of the prior probability distributions are assumed to be known, and the unknown parameters of the reliability growth curve may be estimated with the aid of Bayes Theorem.

Other models considered in the literature may be classified as nonparametric. This approach allows for the estimation of the present system reliability from experimental data without attempting to fit a particular parametric curve. The estimates are usually conservative and projections on future system reliability are generally not possible.

A review of some reliability growth models which have appeared in the literature are given in Appendix A and a bibliography on reliability growth is given in Appendix B.

4. EXAMPLES

To acquaint the reader with the practical applications of reliability growth modeling, several numerical examples shall be presented in this section. The models illustrated by these examples are, also, discussed in Appendix A and appropriate reference to this Appendix will be given for each model considered.

Example 1. The Duane Model (see Appendix A, Model 7) will be illustrated by this example. Briefly, this model assumes that

$$\lambda(T) = KT^{-\alpha},$$

where $\lambda(T)$ is the cumulative failure rate of the system at operating time T and $K > 0$, $0 \leq \alpha \leq 1$ are parameters to be determined before the start of the development program. The example is from "Reliability Planning and Management - RPM" by J. D. Selby and S. G. Miller (see Appendix B, Reference 34).

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A customer generates a requirement to design, develop, and deliver a new avionics equipment which violates no laws of physics, is expected to be within the state of the art, has a lead time of 36 months, and an apportioned MTBF from a higher level system requirement of 150 hours. The reliability program is to be in accordance with MIL-STD-785 with testing per MIL-STD-781 test level F; First Article Configuration Inspection (FACI) and configuration control are required on the first production item.

Viewing these requirements, a contractor first determines a functional implementation plan and equipment schedule based on the technical exhibits, previous experience, equipment complexity, and program planning judgments. Let us assume the case of a responsible contractor who assesses the requirement as being within the state of the art and represents an equipment of 11K parts complexity. Using MIL-STD-217A as a departure point, a prediction of 220 hours is calculated,

made possible by use of screened parts, applied under exacting application and derating criteria. This prediction meets the first RPM criteria by exceeding the minimum 125% of specified requirement.

The schedule milestones are established, based on past development experience, resulting in 15 months for design, 6 additional months for initial hardware manufacture and ambient test, 12 months for evaluation testing, and 3 months for final change documentation and incorporation prior to production FACI and configuration control constraints.

The second and third criteria of the model state that for a new design the initial hardware MTBF will be 10% of that predicted, and reliability growth will follow the Duane postulate. Employing these criteria, a reliability initial estimate and growth requirement based on the specifics of the selected implementation now can be structured. The initial capability for this new system is thus dimensioned as 22 hours, 10% of the predicted MTBF. A growth rate of .5 is planned based upon a comprehensive reliability program executed through competent implementation of MIL-STD-785 and MIL-STD-781. The growth requirements, Figure 1, indicate that compliance can be achieved at 4800 hours of test time.

In implementation planning, optimization of the reliability program about individual program priorities is desirable and required. In this example, the contractor elects to consider optimization in three cases of program structuring: minimum assets, compliant time, and least risk. The objective is clearly to dimension and evaluate the alternatives and activity necessary to achieve required equipment reliability during development and prior to the production and user constraints. Based on a review of test experience with complex systems, it has been established for GE/AES products that 200 hours of test operate time can reasonably be achieved per system month of effort.

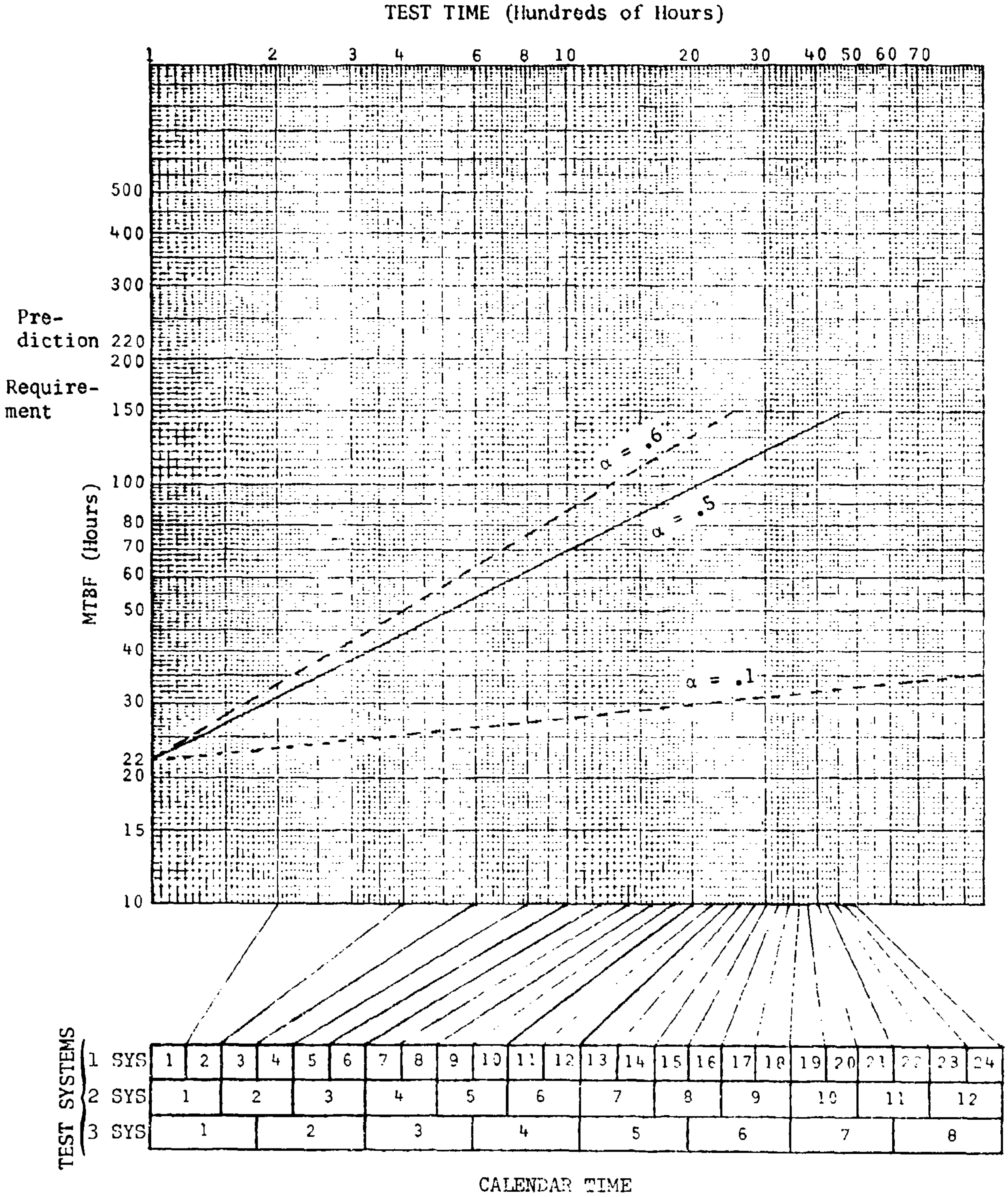

Figure 1. Planning Example

Operating within these bounds, let us now consider the test plan options open to the contractor to develop a compliant product. The first option, minimum assets, requires one system tested continuously for 24 months representing the least number of systems. The second option, a time compliant test, requires two systems tested continuously and concurrently for 12 months. The third option, least risk, requires three systems tested continuously and concurrently for 8 months, accommodating additional time for reaction to contingency including growth of up to 25% in product complexity. This growth reflects the case where an 11K part original estimate grows during detail implementation to a 14K part system.

From these optimizations, the magnitude of the program, kinds of disciplines, and available tradeoffs bounding a successful program are now clear to management.

Example 2. This example and the following two examples illustrate the use of the Gompertz equation considered by Virene (see Appendix A, Model 6). These examples are from Virene's paper entitled, "Reliability Growth and Its Upper Limit" (see Appendix B, Reference 38).

The Gompertz equation is

$$R = ab^{c^t},$$

where a is the (unknown) upper limit approached by the reliability, R, as time $t \rightarrow \infty$, and $0 < b < 1$, $0 < c < 1$ are parameters to be estimated from test data.

The following steps are necessary for estimating a, b, c by Virene's method.

A. Arrange the currently available data, values for t and R, in columns.

B. Calculate values for log R.

C. Divide the column of values for log R into three equal-size groups, each containing n items.

D. Add the values of log R in each group, obtaining sums identified as S_1, S_2, and S_3, starting with the lowest values of log R.

E. Calculate the value for c.

F. Calculate the value for a.

G. Calculate the value for b.

H. Set a value for t, the point to which the projection is to be extended, e.g., 100,000 hours, 30 months, etc.

I. Substitute the values for a, b, and c into the Gompertz equation to obtain the value for R at point t.

The ceiling value of R is a. That is, at any point in time the currently available data can be used to calculate a value for a, which is the highest value that R is likely to reach. Values for a, b, and c are calculated by the following equations:

$$c = \left(\frac{S_2 - S_3}{S_1 - S_2}\right)^{\frac{1}{n}}$$

$$\log a = \frac{1}{n}\left(S_1 - \frac{S_1 - S_2}{1 - c^n}\right)$$

$$\log b = \frac{(S_1 - S_2)\ (1 - c)}{(1 - c^n)^2}$$

Where:

S_1 = Summation of log R in group 1.

S_2 = Summation of log R in group 2.

S_3 = Summation of log R in group 3.

n = Number of previously determined reliability values in a group.

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Suppose that it is contractually required that a certain device have an assessed reliability of 0.90 at the end of a 10-month design and development period. What will be the assessed reliability at the contract completion date, and the maximum achievable reliability without major redesign?

Starting with the available assessment data at the end of five months, convert the reliabilities to percentages and calculate the log R values:

	Time	Reliability Converted to %	
	(t)	(R)	log R
Group 1	0	56.0	1.748
	1	64.0	1.806
Group 2	2	70.5	1.848
	3	76.0	1.881
Group 3	4	80.0	1.903
	5	83.0	1.919

$$S_1 = \begin{array}{r} 1.748 \\ \underline{1.806} \\ 3.554 \end{array} \qquad S_2 = \begin{array}{r} 1.848 \\ \underline{1.881} \\ 3.729 \end{array} \qquad S_3 = \begin{array}{r} 1.903 \\ \underline{1.919} \\ 3.822 \end{array}$$

$$c = \left(\frac{S_2 - S_3}{S_1 - S_2}\right)^{\frac{1}{n}} = \left(\frac{3.729 - 3.822}{3.554 - 3.729}\right)^{\frac{1}{2}} = 0.729$$

$$\log a = \frac{1}{n}\left(S_1 - \frac{S_1 - S_2}{1 - c^n}\right) = 1.964$$

a = 92.0, the upper limit for reliability.

$$\log b = \frac{(S_1 - S_2)\ (1 - c)}{(1 - c^n)^2} = 0.784 - 1$$

$$b = 0.608$$

$$R = ab^{c^t}$$

$$R = 92.0\ (0.608)^{0.729^t}$$

Conclusions:

The upper limit for achievable reliability is indicated by a, which was found to be 92 percent.

When:

$$t = 10 \text{ months, as in this contract}$$

$$R = 92.0\ (0.608)^{0.729^{10}}$$

$$\log R = \log 92.0 + 0.729^{10} \log 0.608$$

$$= 1.964 + 0.0427\ (-0.216)$$

$$= 1.955$$

$R = 90.2$, assessed percent reliability at the end of 10 months.

Since the required reliability is 90 percent, the calculations indicate that the requirement will be met, but just barely. Unless there is a possibility of making a significant improvement by a design change within the remaining 5-month period of the contract, the achievable reliability should be safeguarded by such means as the use of extreme care in material and process controls, inspection procedures, handling methods and transportation arrangements.

Goodness of Fit

Use of the Gompertz equation should depend on its suitability in describing the present data. To determine the goodness of fit of the equation

$$R = 92.0\ (0.608)^{0.729^{t}}$$

calculate reliability values for t = 0, 1, 2, 3, 4, 5 and then compare these values with the presently available reliability values.

If T	R calculated from equation	R data available
0	56.0	56.0
1	64.1	64.0
2	70.6	70.5
3	75.9	76.0
4	80.0	80.0
5	83.2	83.0

The Gompertz curve is a good fit for the data used, since the equation reproduces the available data with less than 1% error.

Example 3. Data from the Lunar Orbiter is tabulated below in Table 1. What will the reliability be in January 1965, and what will be the upper reliability limit without redesign?

The data in Table 1 were based on successive design analysis only, because no test or use data were available at the time the predictions were made. The increase in predicted reliability reflected the increase in detailed knowledge about the design.

Table 1

	Time (t)		Reliability Converted to % (R)	log R
Group 1	0	June 1964	51.0	1.708
	1	July	59.4	1.774
Group 2	2	August	63.8	1.805
	3	September	66.6	1.823
Group 3	4	October	68.8	1.838
	5	November	65.5	1.816

$$S_1 = \begin{array}{r} 1.708 \\ \underline{1.774} \\ 3.482 \end{array} \qquad S_2 = \begin{array}{r} 1.805 \\ \underline{1.823} \\ 3.628 \end{array} \qquad S_3 = \begin{array}{r} 1.838 \\ \underline{1.816} \\ 3.654 \end{array}$$

$$c = \left(\frac{S_2 - S_3}{S_1 - S_2}\right)^{\frac{1}{n}} = \left(\frac{3.628 - 3.654}{3.482 - 3.628}\right)^{\frac{1}{2}} = 0.422$$

$$\log a = \frac{1}{n}\left(S_1 - \frac{S_1 - S_2}{1 - c^n}\right)$$

$$= 1.830.$$

$a = 68.1$, the upper limit for reliability.

It was found on the basis of the first six month's data and the Gompertz Model that the reliability predicted in January 1965 would be 0.680 (which it was) and that the upper limit to the predicted reliability would be 0.681, unless there was a redesign of some modification in the evaluation technique.

Example 4. In the Aerospace Program, the Launch Vehicle History of the Blue Scout indicated 15 successes in 22 trials. Determine the 90% and 80% confidence limits of the reliability of the Blue Scout.

The data at the end of 22 launches are indicated in Table 2.

TABLE 2

No. of Launch	Success	Failure	R Pt. Est. Rel. in %	R 80% Confidence	R 90% Confidence
1		X	0		
2		X	0		
3		X	0		
4	X		25.0		
5		X	20.0		
6		X	16.7		
7	X		28.6		
8	X		37.5	19.9	14.7
9	X		44.4	26.8	21.0
10	X		50.0	32.7	26.7
11	X		54.5	37.8	31.8
12	X		58.2	42.2	36.2
13	X		61.7	46.0	40.2
14	X		64.2	49.4	43.7
15	X		66.7	52.4	46.8
16	X		68.7	55.1	49.6
17		X	65.0	51.5	46.3
18	X		66.7	54.0	48.8
19		X	63.0	50.9	45.9
20	X		65.0	53.1	48.2
21	X		66.7	55.1	50.3
22	X		68.1	57.0	52.3

To determine the lower 90% confidence limit of the reliability of the Launch Vehicle, look at the excerpt from a binomial reliability table for 15 successes in 22 trials and confidence level 0.90. We find that the lower 90% confidence limit of the Launch Vehicle is 0.523. Therefore, we are 90% confident that the true population reliability is at least 0.523. The lower 80% confidence limit is determined in the same way and gives a reliability of at least 0.570.

Based on data from the first 15 launches, we could estimate, using the Gompertz Model, the reliability, with say 80% confidence after 22 launches. (This would be a projected reliability if data beyond 15 launches were not available.) Using data from Table 2, Table 3 is obtained.

Table 3

	No. of Launch L	L = X + 4 X	Reliability in % R	log R
	1		0	
	2		0	
	3		0	
Group 1	4	0	25.0	1.398
Group 1	5	1	20.0	1.301
Group 1	6	2	16.7	1.223
Group 1	7	3	28.6	1.456
Group 2	8	4	37.5	1.574
Group 2	9	5	44.4	1.647
Group 2	10	6	50.0	1.699
Group 2	11	7	54.5	1.736
Group 3	12	8	58.2	1.765
Group 3	13	9	61.7	1.790
Group 3	14	10	64.2	1.807
Group 3	15	11	66.7	1.824
	16	12	68.7	1.837
	17	13	65.0	1.813
	18	14	66.7	1.824
	19	15	63.0	1.799
	20	16	65.0	1.813
	21	17	66.7	1.824
	22	18	68.1	1.833

It is assumed that there is no reliability growth until the 4th launch.

1.398	1.574	1.765
1.301	1.647	1.790
1.223	1.699	1.807
1.456	1.736	1.824
$S_1 = 5.378$	$S_2 = 6.656$	$S_3 = 7.186$

$$c = \left(\frac{S_2 - S_3}{S_1 - S_2}\right)^{\frac{1}{n}} = \left(\frac{6.656-7.186}{5.378-6.656}\right)^{\frac{1}{4}} = 0.802$$

$$\log a = \frac{1}{n}\left(S_1 - \frac{S_1 - S_2}{1 - c^n}\right) = 1.890$$

a = 77.6, the upper limit for reliability.

$$\log b = \frac{(S_1 - S_2)\ (1 - c)}{(1 - c^n)^2} = -0.740 = 0.260 - 1$$

$$b = 0.182$$

$$R = ab^{c^x}$$

$$R = 77.6(0.182)^{0.802^x}$$

After 22 launches, L = 22 and, since L = X + 4 in Table 3, X = 18. (Reliability growth starts when L = 4 or X = 0.)

Then:

$$R = 77.6(0.182)^{0.802^x} = 77.6(0.182)^{0.802^{18}}$$

$$\log R = \log 77.6 + 0.802^{18} \log 0.182$$

$$= 1.886$$

$$R = 76.9\%$$

After 22 launches, the reliability, based on the Gompertz equation, is 76.9%. The actual value in Table 3 is 68.1%.

Example 5. This example illustrates a nonparametric reliability growth model introduced by Barlow and Scheuer (see Appendix A, Model 5). In this model each failure must be classified either as inherent or assignable cause and the development program is conducted in K stages with similar systems being tested within each stage. Further, it is assumed that the probability of an inherent failure, q_0, remains unchanged throughout the development program and the probability, q_i, of an assignable cause failure in the i-th stage does not increase from stage to stage. The example is from "Reliability Growth During a Development Testing Program" by Richard E. Barlow and Ernest M. Scheuer (see Appendix B, Reference 3).

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Suppose that a development testing program yielded the results shown in Table 4. Each stage of sampling, except the last, was terminated when an assignable cause failure occurred. A redesign effort was undertaken to eliminate the cause of failure, so that the test units in the succeeding stage were different from the earlier units but homogeneous in any given stage. It is remarked that this is the defining property of a stage, namely the homogeneity of all test units therein.

Table 4

Stage i	Inherent Failures a_i	Assignable Cause Failures b_i	Successes c_i	Trials $a_i + b_i + c_i$	$\frac{b_i}{b_i + c_i}$
1	0	1	0	1	1
2	0	1	0	1	1
3	0	1	0	1	1
4	1	1	1	3	1/2
5	0	1	4	5	1/5
6	0	1	0	1	1
7	0	1	0	1	1
8	0	1	3	4	1/4
9	9	1	27	37	1/28
TOTALS	10	9	35	54	-

The maximum likelihood estimate of an inherent failure q_o is

$$\hat{q}_o = \sum_{i=1}^{K} a_i / \sum_{i=1}^{K} (a_i + b_i + c_i)$$

and the maximum likelihood estimate of q_i, an assignable cause failure in the i-th stage, is

$$\hat{q}_i = (1-\hat{q}_o)\, b_i/(b_i + c_i)$$

$i=1,\ldots,K$. The $\hat{q}_i$'s are the maximum likelihood estimates of the q_i's in general. Let $\bar{q}_1,\bar{q}_2,\ldots,\bar{q}_K$ denote the maximum likelihood estimates of $q_1,q_2,\ldots,q_K$ subject to the condition that

$$q_1 \geq q_2 \geq \cdots \geq q_K.$$

If

$$\hat{q}_1 \geq \hat{q}_2 \geq \cdots \geq \hat{q}_K,$$

then

$$\bar{q}_i = \hat{q}_i,$$

$i=1,\ldots,K$. If

$$\hat{q}_j < \hat{q}_{j+1}$$

for some j ($j=1,\ldots,K-1$), then combine the observations in the j-th and (j+1)-st stages and compute the maximum likelihood estimates of the q_i's using again the definition of $\hat{q}_i$ given above for the K-1 stages thus formed. This procedure is continued until the estimates of the q_i's form a non-increasing sequence. These estimates are the maximum likelihood estimates of the q_i's subject to

$$q_1 \geq q_2 \geq \cdots \geq q_K.$$

For this example it is found that

$$q_c = .1852$$

$$\bar{q}_1 = \bar{q}_2 = \bar{q}_3 = .3148,$$

$$\bar{q}_4 = .4074,$$

$$\bar{q}_5 = \bar{q}_6 = \bar{q}_7 = .3492,$$

$$\bar{q}_8 = .2037, \text{ and}$$

$$\bar{q}_9 = .0291.$$

Further, the maximum likelihood estimate for r_9, the reliability of the system in its final test configuration, is

$$\bar{r}_9 = 1-\hat{q}_o - \bar{q}_9 = .7857.$$

If no assumption of reliability growth were made - that is, if all test units were (incorrectly) supposed to be homogeneous and if no distinction were made between inherent and assignable cause failures - the estimate of reliability would be

$$\hat{r}_9 = \sum_{i=1}^{9} c_i \Big/ \sum_{i=1}^{9} (a_i + b_i + c_i) = .6481.$$

To find a conservative 100(1-α) percent lower confidence bound for r_9, it is not necessary to distinguish between inherent and assignable cause failures. The procedure is to treat the data as though they were homogeneous, that is, as if no reliability growth were taking place, and use the standard technique to obtain a one-sided lower confidence limit on a binomial parameter having observed s successes in n trials. Using the data in Table 4 it is found that a lower 95 percent confidence bound on r_9 is .53. This is a conservative bound and is the same estimate that would be obtained if no reliability growth was assumed.

Example 6. Barlow, Proschan and Scheuer (see Appendix A, Model 9) considered a reliability growth model similar to the one illustrated in the previous example. The main distinction is that in their model one need not classify failures as inherent or assignable cause. Specifically, their model assumes that a system is being modified during K stages of development. Let p_i be the system reliability at the i-th stage. Barlow, et al, obtained the maximum likelihood estimates of $p_1, p_2, \ldots, p_K$, under the restriction that

$$p_1 \leq p_2 \leq \cdots \leq p_K.$$

Also, a conservative lower confidence bound on p_K was obtained assuming

$$p_K \geq \max_{i<K} p_i.$$

Data consist of x_i successes from n_i observations in stage $i, i=1,\ldots,K$.

To obtain the maximum likelihood estimates of $p_1,\ldots,p_K$ subject to the restriction that $p_1 \leq p_2 \leq \ldots \leq p_K$, first form the ratios $x_1/n_1, x_2/n_2, \ldots, x_K/n_K$. If $x_1/n_1 \leq x_2/n_2 \leq \ldots \leq x_K/n_K$, then x_i/n_i is the MLE $\hat{p}_i$ of p_i. If for some j $(j=1,\ldots,K-1)$, $x_j/n > x_{j+1}/n_{j+1}$, combine the observations in the j-th and (j + 1)-st stages and examine the ratios,

$$x_1/n_1, \ldots, x_{j-1}/n_{j-1}, (x_j + x_{j+1})/(n_j + n_{j+1}),$$

$$x_{j+2}/n_{j+2}, \ldots, x_K/n_K,$$

for the (K - 1) stages thus formed. If these ratios are in nondecreasing order, they constitute the MLE's of $p_1,\ldots,p_K$ with $\hat{p}_j = p_{j+1} = (x_j + x_{j+1})/(n_j + n_{j+1})$. If not, continue the process of combining stages until the ratios are in nondecreasing order. This process need be repeated at most (K - 1) times, and the result is independent of the order in which stages are combined to eliminate reversals in the sequence of ratios.

The example is from "Maximum Likelihood Estimation and Conservative Confidence Interval Procedures in Reliability Growth and Debugging Problems" by Barlow, et al, (see Appendix B, Reference 2).

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The procedure is illustrated by the data in the following table.

Stage (i)	Successes (x_i)	No. of Trials (n_i)	x_i/n_i
1	2	5	.400
2	3	7	.429
3	3	8	.375
4	2	6	.333
5	6	6	1.000

The process of combining stages to get a sequence of non-decreasing ratios is summarized below:

i	x_i	n_i	x_i/n_i	First Combination	Second Combination	Third Combination
1	2	5	.400	.400	.400	
2	3	7	.429			
3	3	8	.375	$\frac{6}{15}$ = .400	$\frac{8}{21}$ = .381	$\frac{10}{26}$ = .385
4	2	6	.333	.333		
5	6	6	1.000	1.000	1.000	1.000

Thus, we obtain the maximum likelihood estimates

$$\hat{p}_1 = \hat{p}_2 = \hat{p}_3 = \hat{p}_4 = .385, \ \hat{p}_5 = 1.000.$$

A conservative $100(1-\alpha)$ percent lower confidence bound on p_K, the reliability of the latest version of the system, is found by treating the data from the K stages of the development program as though they were homogeneous, then applying the standard binomial approach to get a lower confidence bound on a binomial parameter having observed $x = \Sigma_1^K x_i$ successes in $n = \Sigma_1^K n_i$ trials. Thus, to obtain the conservative lower confidence bound on p_K, the stage-by-stage history of the development program is not needed; only the total number of successes and the total number of trials.

In this example, a total of 16 successes were observed in a total of 32 trials. A 95 percent lower confidence bound for a single binomial parameter based on these data is found from binomial tables to be .344. Thus, a conservative 95 percent lower confidence bound for p_5 is .344.

Example 7. The reliability growth model illustrated by this example was, also, considered by Barlow, Proschan, and Scheuer (see Appendix A, Model 10). It is assumed, again, that a system is being modified during K stages of development. Let F_i be the distribution of system life length at the i-th stage of development. Because of reliability growth the maximum likelihood estimates of $F_1(t), F_2(t), \ldots, F_K(t)$ are obtained assuming that

$$\bar{F}_1(t) \leq \bar{F}_2(t) \leq \ldots \leq \bar{F}_K(t)$$

for a fixed $t \geq 0$, where $\bar{F}_i(t) = 1 - F_i(t)$.

Data consist of independent life length observations $X_{i1}, \ldots,$ ${}_{i}$, $i=1,\ldots,K$.

The maximum likelihood estimates (MLE) of $F_1(t), \ldots, F_K(t)$ are obtained as follows. For $i=1,2,\ldots,K$, obtain the empirical distribution function $F_{in_i}(t)$ from $F_{in_i}(t) = m_i(t)/n_i$, where $m_i(t) =$ number of observations among $X_{i1}, X_{i2}, \ldots, X_{in_i}$ not exceeding t. If $F_{1n_1}(t) \geq F_{2n_2}(t) \geq \ldots \geq F_{Kn_K}(t)$, then these constitute MLE's of $F_1(t), F_2(t), \ldots, F_K(t)$ respectively. Suppose, on the contrary, the reversal,

$$\frac{m_i(t)}{n_i} < \frac{m_{i+1}(t)}{n_{i+1}} ,$$

occurs. Then the MLE is obtained by assuming a common value for $F_i(t)$ and $F_{i+1}(t)$. Under this assumption the MLE of this common value is obtained by pooling the observations from the two distributions to obtain

$$\frac{m_i(t) + m_{i+1}(t)}{n_i + n_{i+1}},$$

as the MLE of the common value $F_i(t) = F_{i+1}(t)$. Then examine

$$\frac{m_1(t)}{n_1},\ldots,\frac{m_{i-1}(t)}{n_{i-1}}, \frac{m_i(t) + m_{i+1}(t)}{n_i + n_{i+1}}, \frac{m_i(t) + m_{i+1}(t)}{n_i + n_{i+1}},$$

$$\frac{m_{i+2}(t)}{n_{i+2}},\ldots,\frac{m_K(t)}{n_K}.$$

If these are in decreasing order, they constitute MLE's of $F_1(t),\ldots,F_{i-1}(t),F_i(t),F_{i+1}(t),F_{i+2}(t),\ldots,F_K(t)$. If on the other hand, a reversal exists, pool as before to eliminate the reversal (adding the various $m_i(t)$ involved in the reversal to obtain a new numerator and adding the corresponding n_i to obtain a new denominator). After each reversal has been eliminated, we test the resulting sequence of ratios to see if they are in decreasing order. When finally a sequence of decreasing ratios is obtained, these constitute the MLE's $\hat{F}_1(t)$, $\hat{F}_2(t)$, $\ldots,\hat{F}_K(t)$.

The example is from "Maximum Likelihood Estimation and Conservative Confidence Interval Procedures in Reliability Growth and Debugging Problems" by Barlow, et al, (see Appendix B, Reference 2).

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A development program has two stages of development with four observations in Stage 1 and six observations in Stage 2. The data are:

$$
\begin{array}{llll}
X_{11} = 91 \text{ hours} & & X_{21} = 96 \text{ hours} \\
X_{12} = 54 \text{ hours} & & X_{22} = 49 \text{ hours} \\
X_{13} = 120 \text{ hours} & & X_{23} = 105 \text{ hours} \\
X_{14} = 75 \text{ hours} & & X_{24} = 125 \text{ hours} \\
 & & X_{25} = 101 \text{ hours} \\
 & & X_{26} = 115 \text{ hours}
\end{array}
$$

The MLE's of $F_1(t)$ and $F_2(t)$ are given by

$$
\hat{F}_1(t) = \begin{cases}
0, & t < 49 \\
1/10, & 49 \le t < 54 \\
1/4, & 54 \le t < 75 \\
1/2, & 75 \le t < 91 \\
3/4, & 91 \le t < 115 \\
8/10, & 115 \le t < 120 \\
1, & t \ge 120
\end{cases}
$$

and

$$
\hat{F}_2(t) = \begin{cases}
0, & t < 49 \\
1/10, & 49 \le t < 54 \\
1/6, & 54 \le t < 96 \\
1/3, & 96 \le t < 101 \\
1/2, & 101 \le t < 105 \\
2/3, & 105 \le t < 115 \\
8/10, & 115 \le t < 120 \\
5/6, & 120 \le t < 125 \\
1, & t \ge 125
\end{cases}
$$

Note, again, that the procedure illustrated above yields the MLE for any one, fixed, predetermined value of t, not for the entire distribution function.

5. COMMENTS ON CHOOSING A MODEL

So far the need for reliability growth modeling has been discussed and some background and examples on this subject have been given. The next area of interest is naturally the practical aspects of choosing an appropriate model. Because of the lack of published research in this area, the present section will necessarily address the problem in generalities.

As in any mathematical model, reliability growth models are idealizations and are based on a number of assumptions which vary with the models. With parametric and Bayesian models it may be very difficult or, in fact, impossible to verify the underlying assumptions in practice. Therefore, a problem of prime importance is the robustness of the statistical procedures for the parametric and Bayesian models; i.e. how well will these procedures represent the actual growth process when the underlying assumptions are not entirely satisfied. Unfortunately, sufficient research to answer this question has not been performed. In light of this lack of research, one should be careful when choosing a parametric or Bayesian model since any results obtained from the model may be crucial and expensive to both the user and developer if it does not reasonably represent the growth process.

Another related factor when choosing a parametric reliability growth model is whether statistical procedures are available for estimating and/or determining confidence statements on the unknown parameters. For example, in the simple Lloyd and Lipow model (see Appendix A, Model 1) it is easily shown that a "good" estimator of the failure probability does not, in general, exist.

With little knowledge about the characteristics of the growth process one may wish to use a nonparametric reliability growth model. Of course, one must pay a price for this scarcity of knowledge. The major shortcoming of the nonparametric models is that lower confidence

bounds tend to be overly conservative. The result of this could be unnecessary development cost since important decisions concerning the reliability of the system would usually be based on the lower confidence bound.

One should keep in mind, too, that the growth process is a function of the development effort. If the development effort changes then one may wish to examine the reliability growth model being used to see if it is still realistic. If it is possible that the development effort will change in the future, then one should be cautious when projecting the future reliability based on the present model and past data.

The proper application of reliability growth models may often require a close, continuing relationship between the user, developer and all other interested parties. Everyone involved should realize from the outset that reliability growth modeling is more than a statistical exercise of fitting experimental data to arbitrary mathematical functions. If properly understood and used, reliability growth modeling can be a very important management tool. Otherwise, it may be useless and harmful.

APPENDIX A

A REVIEW OF SOME RELIABILITY GROWTH MODELS

This appendix describes a number of reliability growth models which are currently available. Each model is briefly described including the basic assumptions that were made in deriving the models. Technical references are given for each of these models where a more complete discussion of the model may be found.

Model 1. Lloyd and Lipow (see Appendix B, Reference 28) introduced a reliability growth model for a system which has only one failure mode. For each trial it is assumed that the probability is a constant that the system will fail if the failure mode has not been previously eliminated. If the system does not fail, no corrected action is performed before the next trial. If the system fails, then an attempt is made to remove the failure mode from the system. The probability of successfully removing the failure mode is also assumed to be a constant for each attempt. They show that the system reliability, R_n, on the n-th trial is

$$R_n = 1-Ae^{-C(n-1)}$$

where A and C are parameters.

Model 2. Another reliability growth model was considered by Lloyd and Lipow (see Appendix B, Reference 28) where the development program is conducted in K stages and on the i-th stage a certain number of systems are tested. The reliability growth function considered was

$$R_i = R_\infty-\alpha/i,$$

where R_i is the system reliability during the i-th stage, R_∞ is the ultimate reliability as $i \to \infty$ and $\alpha>0$ is a parameter. Maximum likelihood and least squares estimates of R_∞ and α are given by Lloyd and Lipow

along with a lower confidence limit for R_K.

Model 3. Weiss (see Appendix B, Reference 41) considered a reliability growth model where the mean time to failure of a system with exponential life distribution is increased by removing the observed failure modes. In particular, he showed that when certain conditions hold, the increase of the mean time to failure is approximately at a constant percent per trial. That is, if $\theta(i)$ is the mean time to failure of the system at trial i then $\theta(i)$ may be approximated under certain conditions by

$$\theta(i) = Ae^{Ci},$$

where A and C are parameters. Note that

$$\theta(i+1) = e^{C}\theta(i).$$

The maximum likelihood estimates of A and C are given by Weiss.

Model 4. Wolman (see Appendix B, Reference 42) considered a situation where the system failures are classified according to two types. The first type is termed "inherent cause" and the second type is termed "assignable cause". Inherent cause failures reflect the state-of-the art and may occur on any trial while assignable cause failures may be eliminated by corrective action, never to appear again. Wolman assumed that the number of original assignable cause failures is known and that whenever one of these modes contribute a failure, the mode is removed permanently from the system. Wolman uses a Markov-chain approach to derive the reliability of the system at the n-th trial when the failure probabilities are known.

Model 5. Barlow and Scheuer (see Appendix B, Reference 3) considered a nonparametric model for estimating the reliability of a system during a development program. They assumed that the design and engineering changes do not decrease the system's reliability, but, unlike some other models, they do not fit a prescribed functional form to the

reliability growth. Their model is similar to Wolman's in that each failure must be classified either as inherent or assignable cause.

It is further assumed that the development program is conducted in K stages, with similar systems being tested within each stage. For each stage, the number of inherent failures, the number of assignable cause failures and the number of successes are recorded. In addition, they assumed that the probability of an inherent failure, q_o, remains the same throughout the development program and that the probability of an assignable cause failure, q_i, in the i-th stage does not increase from stage to stage of the development program. The authors obtained the maximum likelihood estimates of q_o and of the q_i's subject to the condition that they be nonincreasing. A conservative lower confidence bound for the reliability of the system in its final configuration was, also, given.

Model 6. Virene (see Appendix B, Reference 38) considered the suitability of the Gompertz equation

$$R = ab^{c^t},$$

$0 < b < 1$, $0 < c < 1$, for reliability growth modeling. In this equation a is the upper limit approached by the reliability R as the development time $t \to \infty$. The parameters a, b and c are unknown. Virene gave estimates of these parameters and demonstrated by examples the application of this model.

Model 7. Duane (see Appendix B, Reference 17) considered a deterministic approach to reliability growth modeling. He analyzed data available for several systems developed by General Electric in an effort to determine if any systematic changes in reliability improvement occurred during the development programs for these systems. His analysis revealed that for these systems, the cumulative failure rate versus cumulative operating hours fell close to a straight line when plotted on log-log

paper. The cumulative failure rate appeared to decrease at approximately the -0.4 or -0.5 power of cumulative operating hours.

The types of systems investigated were of the complex electro-mechanical and mechanical nature. Duane concluded that a line with a slope of -0.5 representing cumulative failure rate as a function of cumulative operating hours on log-log paper would probably be suitable for reflecting reliability growth for similar type systems developed at General Electric.

Mathematically, Duane's failure rate equation may be expressed by

$$\lambda(T) = KT^{-\alpha},$$

$K > 0$, $0 \leq \alpha \leq 1$, where $\lambda(T)$ is the cumulative failure rate of the system at operating time T, and K and α are parameters. It follows then that

$$\lambda(T) = \frac{E(T)}{T}$$

where E(T) is the expected number of failures the system will experience during T units of operation. This yields

$$E(T) = KT^{1-\alpha}.$$

Furthermore, the instantaneous failure rate at T is given by

$$\theta(T) = (1-\alpha)KT^{-\alpha}.$$

For a system with a constant failure rate the mean time between failure (MTBF) of the system at operating time T is

$$M(T) = [\theta(T)]^{-1} = [(1-\alpha)K]^{-1}T^{\alpha}.$$

That is, the change in system MTBF during development is proportional to T^{α}.

With this notation $\alpha = 0.5$ closely represented the types of systems considered by Duane.

Model 8. Pollock (see Appendix B, Reference 32) considered a Bayesian reliability growth model for a system undergoing development. The parameters of the model are assumed to be random variables with appropriate prior distribution functions. Using his results, one may project the system reliability to any time after the start of the development program without data and, also, estimate the system reliability after data have been observed. He further gave precision statements regarding the projection and estimation.

Model 9. Barlow, Proschan and Scheuer (see Appendix B, Reference 2) considered a reliability growth model which assumes that a system is being modified at successive stages of development. At stage i the system reliability (probability of success) is p_i. The model of reliability growth under which one obtains the maximum likelihood estimates of $p_1, p_2, \ldots, p_K$ assumes that

$$p_1 \leq p_2 \leq \cdots \leq p_K.$$

That is, it is required that the system reliability be not degraded from stage to stage of development. No particular mathematical form of growth is imposed on the reliability. In order to obtain a conservative lower confidence bound on p_K, it suffices to require only that

$$p_K \geq \max_{i<K} p_i.$$

That is, it is only necessary that the reliability in the latest stage of development be at least as high as that achieved earlier in the development program.

Data consist of x_i successes in n_i trials in stage i, $i=1,\ldots,K$.

A variation of this model is treated in Barlow and Scheuer (see Model 5). In that model two types of failure, inherent and assignable cause, are distinguished.

Model 10. Another reliability growth model considered by Barlow, Proschan and Scheuer (see Appendix B, Reference 2) assumed that at stage i of development the distribution of system life length is F_i. The model of reliability growth under which the maximum likelihood estimates of $F_1(t), F_2(t), \ldots, F_K(t)$ are obtained, writing

$$\bar{F}_i(t) = 1 - F_i(t)$$

is

$$\bar{F}_1(t) \leq \bar{F}_2(t) \leq \ldots \leq \bar{F}_K(t)$$

for a fixed $t \geq 0$. In order to obtain a conservative upper confidence curve on $F_K(t)$ and thereby, a conservative lower confidence curve on $\bar{F}_K(t)$ for all non-negative values on t, it suffices only to require that

$$\bar{F}_K(t) \geq \max_{i<K} \bar{F}_i(t)$$

for all $t \geq 0$. That is, the probability of system survival beyond any time t in the latest stage of development is at least as high as that achieved earlier in the development program.

Data consist of independent life length observations $X_{i1}, \ldots, X_{in_i}, i=1, \ldots, K$.

Model 11. Barlow, Proschan and Scheuer (see Appendix B, Reference 2), also, considered a reliability growth model which assumes that the system life at the i-th stage of development has increasing failure rate.

Because of improvement from stage to stage

$$r_1(t) \geq r_2(t) \geq \dots \geq r_K(t)$$

for $t \geq 0$, where $r_i(t)$ is the failure rate at time t at the i-th stage of development. That is, for each $t \geq 0$, the probability of system failure in the interval $(t, t+dt)$, given survival till time t, does not increase from stage to stage of the development program.

Given life-length observations, $X_{i1}, X_{i2}, \dots, X_{in}$, the maximum likelihood estimates of $r_1(t), r_2(t), \dots, r_K(t)$ are obtained.

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APPENDIX B

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